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NEW SOLUTIONS OF ANGULAR TEUKOLSKY EQUATION VIA TRANSFORMATION TO HEUNS EQUATION WITH THE APPLICATION OF RATIONAL POLYNOMIAL OF AT MOST DEGREE 2, 3, 4, 5, 6

A.ADERIBIGBE & S. ABAYOMI AKINBODE

Department of Mathematics, Lagos State University, Lagos, Nigeria

ABSTRACT

The perturbation equation of masseless fields for Kerr-de Sitter geometryare written in form of seperable equations as in [17] called the Angular Teukolsky equation. The Angular Teukolsky equation is converted to General Heun's equation with singularities coinciding through some confluent process of one of five singularities. As in [4, 16, and 17] rational polynomials of at most degree six are introduced.

AMS Subject classification: 33XX.

KEYWORDS: Heun Equation, Teukolsky Equation, Type-D Metrics, Polynomial Solutions

1. INTRODUCTION

Teukolsky equation is the consequences of perturbation equation fer Kerr-de Sitter geometry with the separability of angular and radial parts respectively. Carter [18] was the first to discover that the scalar wave function is separable. Other consideration is the $\frac{1}{2}$spin electromagnetic field, gravitational perturbations and gravitino for the Kerr-de Sitter class of geometry.

The Teukolsky equation is applicable in the study of black holes in general. The solutions of the equation are in most cases expressed as series solutions of some specialized functions. This approach has been carried out by so many researchers say Teukol- sky (1973), Breuer et al (1977), Frackerell and Crossman (1977), Leahy and Unruh (1979), Chakrabarti(1984), Siedel(1989), Suzuki et all (1989) just to mention but few. Although Teukolsky equation has five singular points one irregular with four regular points. By some confluent process, these singular points are reduced to four coinciding with the singular points of Heun's equation.

The objective of this work is to obtain polynomial solutions for the derived Tuekolsky equation through its conversion to Heun's equation through rational polynomials of degree at most 2. New solutions in terms of the rational polynomials are obtained.

The paper is organized as follows; The first section deals with the introduction of Teukol- sky equation, the second section deals with the derivation of Teukolsky using the work of [17], the third section has to do with the derivation of Angular Teukolsky and its conversion to Heun's equation and the fourth section has to do with Heun's differential equation and its transformation to hypergeometric differential equation via rational polynomials of at most degree 6. The fifth section gives the various results. All processes follow the works in [4, 16, 17].

2. THE TEUKOLSKY EQUATION [17]

Tekolsky equation was derived using the Kerr (-Newman)-de Sitter geometries

$$ds^2 = -p^2\left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\right) - \frac{\Delta\theta sin^2\theta}{(1+\alpha)^2 p^2}[adt - (r^2 + a^2)d\varphi]^2 + \frac{\Delta_r}{(1-\alpha)^2 p^2}(dt - asin^2\theta d\varphi)^2, \quad \textbf{(2.1)}$$

Where

$$\Delta_r = (r^2 + a^2)\left(1 - \frac{a}{ar^2}r^2\right) - 2Mr + Q^2 =$$

$$-\frac{\alpha}{a^2}(r - r_+)(r - r_-)(r - r'_+)(r - r'_-),$$

$$\Delta_\theta = 1 + acos^2\theta, \alpha = \frac{\Lambda a^2}{3}, \bar{\rho} = r + iacos\theta \text{ And} \rho^2 = \bar{\rho}\bar{\rho}. \quad \textbf{(2.2)}$$

Here Λ is the cosmological constant; M is the mass of the black hole, rMits angular momentum and Qits charge. The electromagnetic field due to the charge of the black hole was given by

$$A_\mu dx^\mu = -\frac{Qr}{(1+\alpha)^2\rho^2}(dt - acos^2\theta d\varphi). \quad \textbf{(2.3)}$$

In particular, the following vectors were adopted as the null tetrad,

$$\iota^\mu = \left(\frac{(1+\alpha)(r^2+a^2)}{\Delta_r}, 1, 0, \frac{a(1+\alpha)}{\Delta_r}\right),$$

$$m^\mu = \frac{1}{2\rho^2}((1+\alpha)(r^2 + a^2), -\Delta_r, 0, a\,(1+\alpha))$$

$$m^\mu = \frac{1}{\bar{p}\sqrt{2\Delta_\theta}}(ia(1+\alpha)sin\theta, 0, \Delta_\theta, \frac{i(1+\alpha)}{sin\theta} m^\mu = m^{*\mu}. \quad \textbf{(2.4)}$$

It was assumed that the time and azimuthal dependence of the fields has the form$e^{-i}(\omega t - m^\mu)$, the tetrad components of derivatives and the electromagnetic field were

$$\iota^\mu = D_0, n^\mu\partial_\mu = \frac{\Delta_r}{2\bar{\rho}}D_0^\dagger, m^\mu\partial_\mu = \frac{\sqrt{\Delta_\theta}}{\sqrt{2}\bar{\rho}}L_0^\dagger,$$

$$m^\mu\partial_\mu = \frac{\sqrt{\Delta_\theta}}{\sqrt{2}\bar{\rho}*}L_0, \iota^\mu A_\mu = -\frac{Qr}{2\rho^2},$$

$$m^\mu A_\mu = \bar{m}^\mu A_\mu = 0 \quad \textbf{(2.5)}$$

Where

$$D_0 = \partial_r - \frac{i(1+\alpha)K}{\Delta_r} + n\frac{\partial_r\Delta_r}{\Delta_r}, D_n^\dagger = \partial_r + \frac{i(1+\alpha)K}{\Delta_r} + n\frac{\partial_r\Delta_r}{\Delta_r}$$

$$L_0 = \partial_\theta - \frac{i(1+\alpha)H}{\Delta_\theta} + n\frac{\partial_\theta(\sqrt{\Delta_\theta}sin\theta)}{\sqrt{\Delta_\theta}sin\theta},$$

$$L_n^\dagger = \partial_\theta - \frac{i(1+\alpha)H}{\Delta_\theta} + n\frac{\partial_\theta(\sqrt{\Delta_\theta}sin\theta)}{\sqrt{\Delta_\theta}sin\theta}. \quad \textbf{(2.6)}$$

With $K = \omega(r^2 + a^2) - am$and $H = -a\omega sin\theta + \frac{m}{sin\theta}$

Using the Newman-Penrose formalism it was showed that perturbation equation in the Kerr-de sitter geometry is separable for massless spin $0, \frac{1}{2}, 1, \frac{3}{2}$and 2 fields. Similarly in the Kerr-Newman-de sitter space those for spin $0, \frac{1}{2}$fields are also separable. The separated equations for fields with spin s and charge e were given by

$$[\sqrt{\Delta_\theta} L^\dagger_{n-s} \sqrt{\Delta_\theta} L_s$$

$$-2(1+\alpha)(2s-1)a\omega cos\theta - 2\alpha(s-1)(2s-1)cos^2\theta + \lambda]S_s(\theta) = 0$$

$$[\Delta_r D_1 D_s^\dagger + 2(1+\alpha)(2s-1)i\omega - \frac{2\alpha}{a^2}(s-1)(2s-1)$$

$$+\frac{-2(1+\alpha)eQKr + iseQr\partial_r\partial_r}{\Delta_r} - 2iseQ - \lambda]R_s(r) = 0 \quad \textbf{(2.7)}$$

3. TRANSFORMATION OF TEUKOLSKY EQUATION TO HEUN'S EQUATION [17]

It was shown in [17] that the Teukolsky equations after separation can be transformed to the Heun's equation by factoring out a single regular singularity.

3.1 Angular Teukolsky Equation

From (2.7), the angular Teukolsky equation after separation was shown to be

$$\{\frac{d}{dx}\left[(1+\alpha x^2)(1-x)\frac{d}{dx}\right] + \lambda - s(1-\alpha) + \frac{(1+\alpha)^2}{\alpha}\xi^2 - 2\alpha x^2$$

$$+\frac{1+\alpha}{1+\alpha x^2}[s(\alpha m - (1+\alpha)\xi)x - \frac{(1+\alpha)^2}{\alpha}\xi^2 - 2m(1+\alpha)\xi + s^2$$

$$-\frac{(1-\alpha)^2 m^2}{(1+\alpha x^2)(1-x^2)} - \frac{(1-\alpha)(s^2+2smx)}{1-x^2}\}S(x) = 0, \quad \textbf{(3.1)}$$

Where $x = cos\theta$ and $\xi = a\omega$. This equation has five regular singularities as $\pm 1, \pm\frac{l}{\sqrt{\alpha}}and\infty$. It was also noted that the angular equation has no independence on M ad Q.by choosing the variable z such as

$$z = \frac{1-\frac{l}{\sqrt{\alpha}}}{2}\frac{x+1}{x-\frac{l}{\sqrt{\alpha}}}$$

Then (3.1) takes the following form,

$$\left\{\frac{d^2}{dz^2} + \left[\frac{1}{z} + \frac{1}{z-1} + \frac{1}{z-z_s} - \frac{2}{z-z_\infty}\right]\frac{d}{dz}\right\}$$

$$1\left(\frac{m-s}{2}\right)^2\frac{1}{z^2} - \left(\frac{m-s}{2}\right)^2\frac{1}{(z-1)^2} + \left(\frac{1+\alpha}{\sqrt{\alpha}}\xi - \frac{\sqrt{\alpha}m + \iota s}{2}\right)^2\frac{1}{(z-z_s)^2} +$$

$$\frac{2}{(z-z_\infty)^2} + [-\frac{m^2}{2}\left(1 + \frac{4\alpha}{(1+\iota\sqrt{\alpha})^2}\right) + \frac{s^2}{2}\left(\frac{1-\iota\sqrt{\alpha}}{1+\iota\sqrt{\alpha}}\right)^2 + \frac{2\iota ms\sqrt{\alpha}}{1-\iota\sqrt{\alpha}}$$

$$+\frac{\lambda - s(1-\alpha) - 2\alpha + 2(1+\alpha)(m+s)\xi}{(1+\iota\sqrt{\alpha})^2}]\frac{1}{z}$$

$$+[\frac{m^2}{2}(1 + \frac{4\alpha}{(1+\iota\sqrt{\alpha})^2}) - \frac{s^2}{2}\left(\frac{1+\iota\sqrt{\alpha}}{1-\iota\sqrt{\alpha}}\right)^2 + \frac{2\iota ms\sqrt{\alpha}}{1-\iota\sqrt{\alpha}}$$

$$+\frac{\lambda-s(1-\alpha)-2\alpha+2(1+\alpha)(m-s)\xi}{(1+\iota\sqrt{\alpha})^2}]\frac{1}{z-1}$$

$$-m^2\frac{2\iota\sqrt{\alpha m}}{(1+\alpha)^2}-s^2\frac{\sqrt{\alpha}(1-\alpha)}{(1+\alpha)^2}-ms\frac{\alpha}{1+\alpha}+\frac{\iota\sqrt{\alpha}(\lambda-s(1-\alpha)+2)}{(1+\alpha)^2}$$

$$+\frac{2\iota\sqrt{\alpha m}+(\alpha-1)s)\xi}{\alpha+1}]\frac{4}{z-z_s}-\frac{8\iota\sqrt{\alpha}}{1-\alpha}\frac{1}{z-z_\infty}\}S(z)=0, \quad (3.2)$$

Where $A_1=\frac{[m-s]}{2}, A_2=\frac{[m-s]}{2}$, and $A_2=\pm\frac{\iota}{2}(\frac{1+\alpha}{\sqrt{\alpha\xi}}-\sqrt{\alpha m}-\iota s)$. Now $f(z)$satisfies the equation

$$\{\frac{d}{dz^2}+[\frac{2A_1+1}{z}+\frac{2A_2+1}{z-1}+\frac{2A_3+1}{z-z_s}]\frac{d}{dz}+\frac{\rho\pm z+u}{z(z-1)(z-z_s)}\}f(z)=0, \quad (3.3)$$

Where

$$\rho\pm=A_1+A_2+A_3\pm A_3^*+1$$

$$u=\frac{-\iota}{4\sqrt{\alpha}}\{\lambda-s(1-\alpha)-2\alpha+2(1+\alpha)(m+s)\xi-\left(1+\iota\sqrt{\alpha}\right)^2(2A_1A_2+A_1+A_2)$$

$$-4\iota\sqrt{\alpha}(2A_1A_3+A_1+A_3)\frac{m^3}{2}[4a+1+(\iota\sqrt{\alpha})^2]$$

$$+\frac{s^3}{2}(1-\iota\sqrt{\alpha})^2+2\iota ms\sqrt{\alpha}(1+\sqrt{\alpha})\}.$$

Equation (3.3) is called the Heun's equation which has four regular singularities. The $f(z)$ is determined by requiring non-singular behaviors at z =0 and1. We can take either one of signs of $A_3$to find the solution $S(z)$in terms of solution of Heun's differential equation.

Every homogenous linear second order differential equation with four regular singularities can be transformed into (3.3) with the assumption that $2A_1+1=\gamma, 2A_2+1=\delta, 2A_3+1=\epsilon, \rho\pm=\alpha\beta$ and $u=q, z=t$ and, $z_s$as defined above, and read as

$$\frac{d^2u}{dt^2}+\left(\frac{\gamma}{t}+\frac{\delta}{t-1}+\frac{\epsilon}{t-d}\right)\frac{du}{dt}+\frac{\alpha\beta t-q}{t(t-1)(t-d)}u=0, \quad (3.4)$$

Where$\{\alpha,\beta,\gamma,\epsilon,d,q\}(d\neq 0,1)$ are parameters, generally complex and arbitrary, linked by FUSCHAIN constraint$\alpha+\beta+1=\gamma+\delta+\epsilon$. This equation has four regular singular points at $\{0,1,a,\infty\}$,, with the exponents of these singular being respectively, $\{0,1,-\gamma\},\{0,1,-\delta\},\{0,1,-\epsilon\}and\{\alpha,\beta\}$. The equation (3.4) is called Heun's equation.

4. HEUN TO HYPERGEOMETRIC IN KUMMER FUNCTION [4]

In this section we show that the Heun's equation (3.4), derived from transformation of Angular Teukolsky Equation can be solved via some polynomial transformation by taken the derivative of the initial solution in relation to a hypergeometric function.

To achieve this objective, let $\mathcal{H}_n(a,q;\alpha,\beta,\gamma,\delta,\epsilon;x)$be the analytic solution of (3.4) around $x=0$ and normalized by$\mathcal{H}_n(0)=1$, we seek to answer the following questions

(i)When is $\mathcal{H}(x)$reducible to some hypergeometric equation ${}_2F_1$?

- (ii) When is $D\mathcal{H}(x)$a good choice of parameters?

Maier [4] in 2005 solved the problem (i) in full generality from the following theorem, enlarging the work of Kuiken [16]

Theorem 4.1*If the Heun's equation parameter values* $(a, q, \alpha, \beta, \gamma, \delta$*are such that the H eun 's equation is not trivial*$q \neq 0$ *or* $\alpha\beta \neq 0$, *and all four of* $t = 0, 1, a, \infty$*are singu- lar points, then there are only seven noncomposite no prefactorHeun-to-hypergeometric transformations, up to isomorphism. These seven transformations involve polynomial maps of degree 2, 3, 4, 3, 4, 5,6respectively. A representative list gives*

- $\mathcal{H}n(2, \alpha\beta, \alpha, \beta, \gamma, \alpha + \beta - 2\gamma + 1; t) = {}_2F_1\left(\frac{\alpha}{3}, \frac{\beta}{3}; \frac{1}{2}; 1(1-t)^2\right).$ **(4.1)**

- $\mathcal{H}n\left(4, \alpha\beta, \alpha, \beta\frac{1}{2}, \frac{2(\alpha+\beta)}{3}; t\right) = {}_2F_1\left(\frac{\alpha}{2}, \frac{\beta}{2}, \gamma, 1-(1-t)^2(1-\frac{t}{4}\right).$ **(4.2)**

- $\mathcal{H}n\left(2, \alpha\beta, \alpha, \beta, \frac{\alpha+\beta}{2}; t\right) = {}_2F_1\left(\frac{\alpha}{4}, \frac{\beta}{4}; \frac{\alpha+\beta+2}{4}; 1-4[t)2+t)-\frac{1}{2}]^2\right).$ **(4.3)**

- $\mathcal{H}n(\frac{1}{2}+i\frac{\sqrt{3}}{2}, \alpha\beta, \left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\alpha, \beta, \frac{\alpha+\beta+1}{3}; \frac{\alpha+\beta+1}{3}; t) = {}_2F_1\left(\frac{\beta}{3}, \frac{\alpha}{3}; \frac{\alpha+\beta+2}{3}; 1-\left(1-\frac{t}{\frac{1}{2}+\frac{i\sqrt{3}}{6}}\right)^3\right).$ **(4.4)**

- $\mathcal{H}n\left(\frac{1}{2}+i\frac{5\sqrt{2}}{4}, \alpha\left(\frac{2}{3}-\alpha\right)\left(\frac{1}{2}+\frac{\sqrt{2}}{4}\right); \alpha\frac{2}{3}-\alpha, \frac{1}{2}, \frac{1}{2}; t\right) = {}_2F_1\left(\frac{\alpha}{4}, \frac{2-3\alpha}{12}; \frac{1}{2}; 1-\left(1-\frac{4t}{2+2i\sqrt{2}}\right)^3\left(1-\frac{4t}{2+2i\sqrt{2}}\right)\right)$ **(4.5)**

- $\mathcal{H}n\left(\frac{1}{2}+i\frac{11\sqrt{15}}{90}, \alpha\left(\frac{5}{6}-\alpha\right)\left(\frac{1}{2}+\frac{\sqrt{15}}{18}\right); \alpha\frac{5}{6}-\alpha, \frac{2}{3}, \frac{2}{3}; t\right) =$

  ${}_2F_1\left(\frac{\alpha}{5}, \frac{5-6\alpha}{30}; \frac{2}{3}; -\frac{2025}{64}i\sqrt{15t}(-1+t)\left(1-\frac{18t-9-i\sqrt{15t}}{18}\right)^3\right)$ **(4.6)**

- $\mathcal{H}n\left(\frac{1}{2}+i\frac{5\sqrt{2}}{4}, \alpha\left(\frac{2}{3}-\alpha\right)\left(\frac{1}{2}+\frac{\sqrt{2}}{4}\right); \alpha\frac{2}{3}-\alpha, \frac{2}{3}, \frac{2}{3}; t\right) = {}_2F_1\left(\frac{\alpha}{6}, \frac{1}{6}-\frac{\alpha}{6}, \frac{2}{3}; 1-\{[1-\frac{t}{\frac{1}{2}+\frac{i\sqrt{3}}{6}}]^3-\frac{1}{2}\}^2\right).$ **(4.7)**

5. MAIN RESULTS

Let us notice that the six quadratic transforms of Kuiken reduces to one, the others being" composite" resulting from the known quadratic transforms of the hypergeometric function${}_2F_1$. Also the four last cases, the singular point $x = \alpha$ is located in the complex plane. Applying the derivative property of the hypergeometric functions:

$$\frac{d}{dx}{}_2F_1\left(a, b; c; x = R(t)\right) = \frac{ab}{c}R'(t){}_2F_1\left(a+1, b+1; C+1'x = R(t)\right) \quad \textbf{(5.1)}$$

For instance, the derivative of the second degree transformation i) generates another ${}_2F_1$with a linear prefactor

$$\frac{d}{dx}{}_2F_1\left(\frac{\beta}{2}, \frac{\alpha}{2}; \gamma; 1-(1-t)^2\right) = -\frac{\beta\alpha}{2\gamma}(-1+t){}_2F_1\frac{\beta+2}{2}, \frac{\alpha+2}{2}; \gamma; 2t-t^2), \quad \textbf{(5.2)}$$

and the pull back operator with $\mathcal{H}n = S_n$to reflect. the solution of (3.1), gives

$$\frac{d}{dt}S_n(2, \alpha\beta, \alpha, \beta, \gamma, \alpha+\beta-2\gamma+1; t) = S_n(2, (\alpha+2)(\beta, +2), \beta, +2, \alpha+2, \gamma+1, \alpha+\beta-2\gamma+3; t). \quad \textbf{(5.3)}$$

$$\frac{d}{dt}S_n\left(4,\alpha\beta,\alpha,\beta,\frac{1}{2},\frac{2(\alpha+\beta)}{3};t\right)=\frac{(3-4t+t^2)}{6}\times S_n\left(4,\alpha+3),(\beta+3);\ \alpha+3;\beta+3,\frac{2}{3};t\right) \tag{5.4}$$

$$\frac{d}{dt}S_n\left(2,\alpha\beta;\alpha,\beta,\frac{\alpha+\beta+2}{4},\frac{\alpha+\beta}{2};t\right)=\frac{2\alpha\beta}{\alpha+\beta+2}(2t^2-4t+1)(-1+t)\times$$

$$S_n\left(2,\alpha+4)(\beta+4);\alpha+4,\beta+4,\frac{\alpha+\beta+6}{4},\frac{\alpha+\beta+8}{2};t\right). \tag{5.5}$$

$$\frac{d}{dt}S_n\left(\frac{1}{2}+i\frac{\sqrt{3}}{2},\alpha\beta(\frac{1}{2}+i\frac{\sqrt{3}}{2},\alpha,\beta,\frac{\alpha+\beta+1}{3};\frac{\alpha+\beta+1}{3};t\right)$$

$$=6\alpha\beta\left(-3-i\sqrt{3}+6t\right)^2(\alpha+\beta+1)^{-1}(3+i\sqrt{3})^{-3}\times$$

$$S_n\left(\frac{1}{2}+i\frac{\sqrt{3}}{2},(\alpha+1)(\beta+1)(\frac{1}{2}+i\frac{\sqrt{3}}{2},(\alpha+1)(\beta+1),\frac{\alpha+\beta+1}{3};\frac{\alpha+\beta+1}{3};t\right). \tag{5.6}$$

$$\frac{d}{dt}S_n\left(\frac{1}{2}+i\frac{5\sqrt{2}}{4},\alpha(\frac{2}{3}-\alpha)(\frac{1}{2}+i\frac{\sqrt{2}}{4};\alpha,\frac{2}{3}-\alpha,\frac{1}{2},\frac{1}{2};t\right)=$$

$$\frac{4\alpha(-2+3\alpha\left(-2-i\sqrt{2}+4t\right)^2(-1-2i\sqrt{2}+2t}{3\left(2+i\sqrt{2}\right)^3(2+5i\sqrt{2})}$$

$$S_n\left(\frac{1}{2}+i\frac{5\sqrt{2}}{4},-(\alpha+4)(\frac{10}{3}-\alpha)(\frac{1}{2}+i\frac{\sqrt{2}}{4};\alpha+4,\frac{14}{3}-\alpha,\frac{3}{2},\frac{3}{2};t\right) \tag{5.7}$$

$$\frac{d}{dt}S_n\left(\frac{1}{2}+i\frac{11\sqrt{15}}{90},\alpha(\frac{5}{6}-\alpha)(\frac{1}{2}+i\frac{\sqrt{15}}{18};\alpha,\frac{5}{6}-\alpha,\frac{2}{3},\frac{2}{3};t\right)=$$

$$\frac{\sqrt{15}}{18432}i\alpha(-5+6\alpha)\left(18t-9-i\sqrt{15}\right)^2(-90t+9+i\sqrt{15}+90t^2-2it\sqrt{15}\times$$

$$S_n\left(\frac{1}{2}+i\frac{11\sqrt{15}}{90},(\alpha+5)(-\frac{25}{6}-\alpha)(\frac{1}{2}+i\frac{\sqrt{15}}{18};\alpha+5,-\frac{25}{6}-\alpha,\frac{5}{3},\frac{5}{3};t\right). \tag{5.8}$$

$$\frac{d}{dt}S_n\left(\frac{1}{2}+i\frac{\sqrt{3}}{2},\alpha(1-\alpha)(\frac{1}{2}+i\frac{\sqrt{3}}{6};\alpha;1-\alpha,\frac{2}{3},\frac{2}{3};t\right)=t\frac{\alpha(1-\alpha)\left[\left(\frac{1}{2}\pm i\frac{\sqrt{3}}{2}-t\right)^3\frac{1}{2}\right]\left[\frac{1}{2}\pm i\frac{\sqrt{3}}{2}-t\right]^2}{\left(\frac{1}{2}\pm i\frac{\sqrt{3}}{2}\right)^6}$$

$$S_n\left(\frac{1}{2}\pm i\frac{\sqrt{3}}{2},(\alpha+1)(-\alpha)(\frac{1}{2}+i\frac{\sqrt{3}}{6};\alpha+1;-\alpha,\frac{5}{3},\frac{5}{3};t\right) \tag{5.9}$$

6. CONCLUDING REMARKS AND SUGGESTIONS

In this paper, we have shown that the solutions of the derived Angular Teukolsky equation transformed to Heun's equation could be obtained in form of Heun's functions via polynomials of at most degree six transformations. The new solutions obtained were as a result of the work of [4]. The integral operator application is also in progress.

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